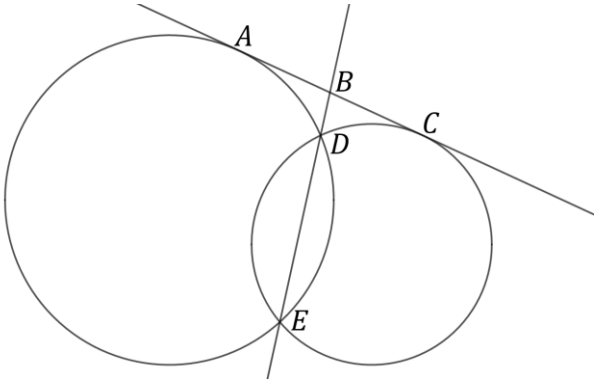
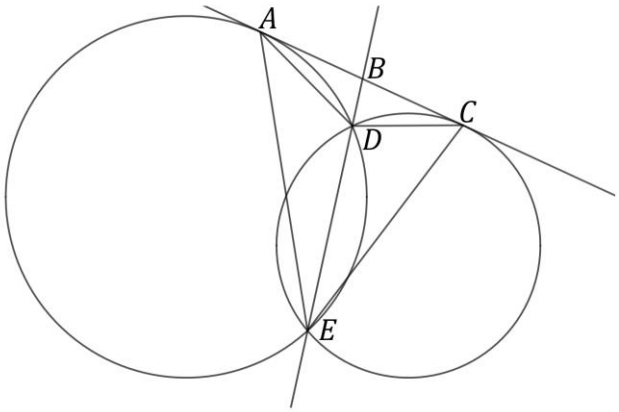
A Geometry Proof

In the diagram below AC is a tangent to both circles. The circles intersect at D and E. BDE is a straight line.

Prove that $AB = BC$.



Draw the line segments AE, AD, DC and CE.



Triangles BAD and BEA are similar since angle BAD is common to both and angle $BAD =$ angle AED, by the alternate segment theorem.

It follows that $\frac{AB}{BE} = \frac{BD}{AB} \Rightarrow AB^2 = BD \times BE$.

Similarly, triangles BCD and BEC are similar therefore $BC^2 = BD \times BE$.

$AB = BC$.