AQA FP1 June 2017

2 The equation $5x^2 + px + q = 0$, where p and q are constants, has roots α and $\alpha + 4$.

(a) Show that $p^2 = 20q + 400$.

[4 marks]

(b) A quadratic equation has roots α^2 and $(\alpha + 4)^2$.

(i) Find this quadratic equation, giving your answer in terms of q.

[3 marks]

(ii) Hence, or otherwise, given that the roots of this quadratic equation are equal, find the value of q.

[2 marks]

The sum of the roots is $2\alpha + 4 = -\frac{p}{5} \Rightarrow \alpha = -\frac{p}{10} - 2$

The product of the roots is $\alpha^2 + 4\alpha = \frac{q}{5} \Rightarrow$

$\left(-\frac{p}{10} - 2\right)^2 + 4\left(-\frac{p}{10} - 2\right) = \frac{q}{5} \Rightarrow \frac{p^2}{100} + \frac{4p}{10} + 4 - \frac{4p}{10} - 8 = \frac{q}{5} \Rightarrow$

$\frac{p^2}{100} - 4 = \frac{q}{5} \Rightarrow p^2 - 400 = 20q \Rightarrow \boldsymbol{p^2 = 20q + 400}$

If α and $\alpha + 4$ are the roots of $5x^2 + px + q = 0$ then α^2 and $(\alpha + 4)^2$ are the roots of $5u + p\sqrt{u} + q = 0$

Rearranging gives $5u + q = -p\sqrt{u} \Rightarrow 25u^2 + 10qu + q^2 = p^2u \Rightarrow$

$25u^2 + (10q - p^2)u + q^2 = 0 \Rightarrow \boldsymbol{25u^2 - (10q + 400)u + q^2 = 0}$

The sum of the roots is $\frac{10q+400}{25}$ and each root is $\frac{5q+200}{25} = \frac{q}{5} + 8$

The product of the roots is $\left(\frac{q}{5} + 8\right)^2 = \frac{q^2}{25} \Rightarrow \frac{q}{5} + 8 = \pm\frac{q}{5} \Rightarrow \frac{q}{5} + 8 = -\frac{q}{5} \Rightarrow$

$\frac{2q}{5} = -8 \Rightarrow q = -20$