AQA S1 June 2016 Question 5

5 Still mineral water is supplied in 1.5-litre bottles. The actual volume, X millilitres, in a bottle may be modelled by a normal distribution with mean $\mu = 1525$ and standard deviation $\sigma = 9.6$.

(a) Determine the probability that the volume of water in a randomly selected bottle is:

(i) less than 1540 ml;

(ii) more than 1535 ml;

(iii) between 1515 ml and 1540 ml;

(iv) not 1500 ml.

[7 marks]

(b) The supplier requires that only 10 per cent of bottles should contain more than 1535 ml of water.

Assuming that there has been no change in the value of σ, calculate the reduction in the value of μ in order to satisfy this requirement. Give your answer to one decimal place.

[4 marks]

(c) Sparkling spring water is supplied in packs of six 0.5-litre bottles. The actual volume in a bottle may be modelled by a normal distribution with mean 508.5 ml and standard deviation 3.5 ml.

Stating a necessary assumption, determine the probability that:

(i) the volume of water in **each** of the 6 bottles from a randomly selected pack is more than 505 ml;

(ii) the **mean** volume of water in the 6 bottles from a randomly selected pack is more than 505 ml.

[7 marks]

$X \sim N(1525, 9.6^2)$

a) i) $P(X < 1540) = P\left(Z < \frac{1540-1525}{9.6}\right) = P(Z < 1.5625) = 0.9409$

ii) $P(X > 1535) = P\left(Z > \frac{1535-1525}{9.6}\right) = P(Z > 1.041\dot{6})$
$= 1 - P(Z < 1.041\dot{6}) = 1 - 0.8512 = 0.1488$

iii) $P(1515 < X < 1540) = P(X < 1540) - P(X < 1515)$
$= P(X < 1540) - P(X > 1535) = 0.9409 - 0.1488 = 0.7921$

iv) $P(X \neq 1500) = 1$

b) Require $P(X > 1535) = 0.1$.
$\Rightarrow\ P(X < 1535) = 0.9$
$\Rightarrow P\left(Z < \frac{1535-\mu}{9.6}\right) = 0.9$

From table 4 of the AQA statistical tables $P(Z \leq 1.2816) = 0.9$

It is therefore required that $\frac{1535-\mu}{9.6} = 1.2816$.

$\mu = 1535 - 1.2816 \times 9.6 = 1522.7$

The required reduction is $1525 - 1522.7 = 2.3$ ml.

c) i) Let B be the volume of water in a bottle. $B \sim N(508.5, 3.5^2)$

The assumption is that the six bottles in the pack are a random sample from the above distribution.

$P(B > 505) = P\left(Z > \frac{505-508.5}{3.5}\right) = P(Z > -1) = P(Z < 1) = 0.8413$
The probability that all six bottles contain more than 505 ml is $0.8413^6 = 0.3546$

ii) The distribution of the sample mean is $\bar{B} \sim N\left(508.5, \frac{3.5^2}{6}\right)$

$P(\bar{B} > 505) = P\left(Z > \frac{505-508.5}{\frac{3.5}{\sqrt{6}}}\right) = P(Z > -\sqrt{6}) = P(Z < \sqrt{6}) = 0.9928$