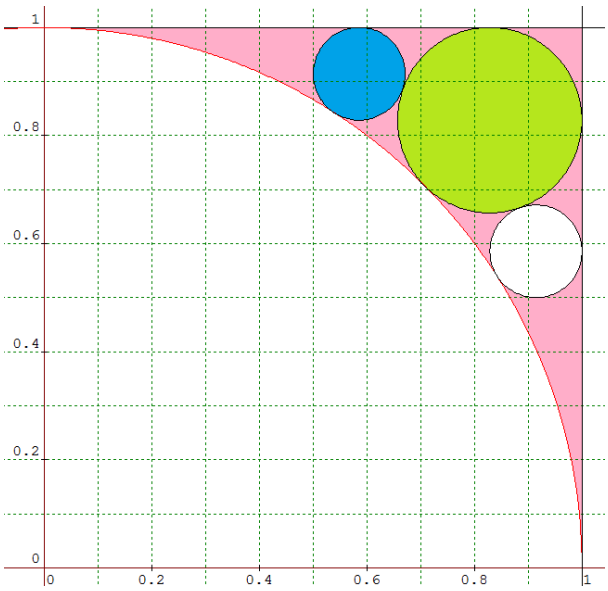
An Area Problem



Find the pink shaded area.

Let the radii of the green and blue circles be R and r.

By considering the diagonal of the square it can be seen that $1 + R + \sqrt{2}R = \sqrt{2}$ and so it follows that $R = \frac{\sqrt{2}-1}{\sqrt{2}+1} = 3 - 2\sqrt{2}$.

Let the centre of the blue circle be $(a, 1 - r)$.

By Pythagoras' theorem $(1 + r)^2 = a^2 + (1 - r)^2 \Longrightarrow a^2 = 4r$.

Considering the distance between the centres of the green circle and the blue circle, we find that

$(R - r)^2 + (1 - a - R)^2 = (R + r)^2$.

Expanding, simplifying and substituting gives $a^2 - 2a + \sqrt{2} - 1 = 0$.

$a = 2 - \sqrt{2}$

$r = \frac{3}{2} - \sqrt{2} = \frac{R}{2}$

The area shaded pink is therefore $1 - \frac{\pi}{4} - \frac{3\pi}{2}\left(3 - 2\sqrt{2}\right)^2 = 1 - \pi\left(\frac{103}{4} - 18\sqrt{2}\right) \approx 0.07588$