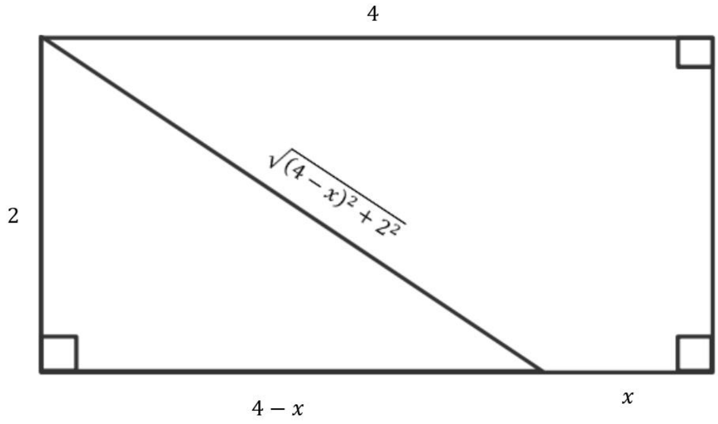
Bob reaches the corner of a large rectangular area of heathland, which is 4 km long and 2 km wide. He needs to reach the opposite corner as quickly as possible. He can walk along the edge at 5 km/h and across the heath at 4 km/h. Bob decides to walk some distance along the edge and then cross the heath. How far, to the nearest, metre should Bob walk along the edge? What is the minimum time required to cross the heath?



Let the distance travelled along the edge be x km.

By Pythagoras' theorem the distance travelled across the heath is $\sqrt{(4-x)^2+2^2} = \sqrt{x^2-8x+20}$

The time taken for the first part of the journey is $\frac{x\text{ km}}{5\text{ km/h}} = \frac{x}{5}$ h.

The time taken for the second part of the journey is $\frac{\sqrt{x^2-8x+20}\text{ km}}{4\text{ km/h}} = \frac{\sqrt{x^2-8x+20}}{4}$ h.

The total time required is $\left(\frac{x}{5}+\frac{\sqrt{x^2-8x+20}}{4}\right)$ h.

Let $T(x) = \frac{x}{5}+\frac{\sqrt{x^2-8x+20}}{4}$ be the total time, in hours, required to complete the journey.

$T'(x) = \frac{1}{5}+\frac{1}{4}\times\frac{x-4}{\sqrt{x^2-8x+20}}$

For the minimum time $T'(x) = 0$

$T'(x) = 0 \Rightarrow \frac{1}{5}+\frac{1}{4}\times\frac{x-4}{\sqrt{x^2-8x+20}} = 0$

$\Rightarrow 4+\frac{5x-20}{\sqrt{x^2-8x+20}} = 0 \Rightarrow 5x-20 = -4\sqrt{x^2-8x+20}$

$\Rightarrow 25x^2-200x+400 = 16x^2-128x+320$

$\Rightarrow 9x^2-72x+80 = 0$

$\Rightarrow (3x-20)(3x-4) = 0$

$\Rightarrow x = \frac{4}{3}$

The distance travelled along the edge, to the nearest metre, is 1333 m.

The time required is $T\left(\frac{4}{3}\right) = \frac{4/3}{5}+\frac{\sqrt{\left(\frac{4}{3}\right)^2-8\left(\frac{4}{3}\right)+20}}{4} = 1.1$ h or 1 hour and 6 minutes.