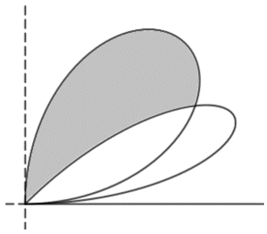
Core Pure 2 Polar Coordinates

The diagram shows the curves with equations $r = 3\sin 4\theta$ and $r = 3\sin 2\theta$ for $0 \le \theta \le \frac{\pi}{2}$.

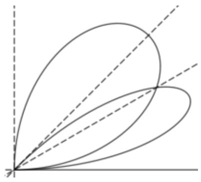
Find the area of the region shaded in the diagram.



At the intersections of the curves $\sin 4\theta = \sin 2\theta$

$2\sin 2\theta\cos 2\theta = \sin 2\theta \Rightarrow \sin 2\theta = 0$ or $\cos 2\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$ or $\theta = \frac{\pi}{2}$

For the curve $r = 3\sin 4\theta$, $r = 0$ when $\theta = \frac{\pi}{4}$.



$$\text{Area} = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (3\sin 2\theta)^2 d\theta - \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (3\sin 4\theta)^2 d\theta$$

$$= \frac{9}{4}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 1 - \cos 4\theta \, d\theta - \frac{9}{4}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 1 - \cos 8\theta \, d\theta$$

$$= \frac{9}{4}\left[\theta - \frac{1}{4}\sin 4\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} - \frac{9}{4}\left[\theta - \frac{1}{8}\sin 8\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$$

$$= \frac{9}{4}\left(\frac{\pi}{2} - \frac{\pi}{6} + \frac{1}{4}\sin\frac{2\pi}{3}\right) - \frac{9}{4}\left(\frac{\pi}{4} - \frac{\pi}{6} + \frac{1}{8}\sin\frac{4\pi}{3}\right)$$

$$= \frac{3\pi}{4} + \frac{9\sqrt{3}}{32} - \frac{3\pi}{16} + \frac{9\sqrt{3}}{64}$$

$$= \frac{9\pi}{16} + \frac{27\sqrt{3}}{64}$$