9. The amount of an antibiotic in the bloodstream, from a given dose, is modelled by the formula

$$x = D\mathrm{e}^{-0.2t}$$

where x is the amount of the antibiotic in the bloodstream in milligrams, D is the dose given in milligrams and t is the time in hours after the antibiotic has been given.

A first dose of 15 mg of the antibiotic is given.

(a) Use the model to find the amount of the antibiotic in the bloodstream 4 hours after the dose is given. Give your answer in mg to 3 decimal places.

(2)

A second dose of 15 mg is given 5 hours after the first dose has been given. Using the same model for the second dose,

(b) show that the **total** amount of the antibiotic in the bloodstream 2 hours after the second dose is given is 13.754 mg to 3 decimal places.

(2)

No more doses of the antibiotic are given. At time T hours after the second dose is given, the total amount of the antibiotic in the bloodstream is 7.5 mg.

(c) Show that $T = a\ln\left(b + \frac{b}{\mathrm{e}}\right)$, where a and b are integers to be determined.

(4)

a) $x(4) = 15e^{-0.2\times4} = 6.740$ mg

b) $x(5) = 15e^{-0.2\times5} = 5.518$ mg

Starting from the second dose $x(2) = (15 + 5.518)e^{-0.2\times2} = 13.754$ mg

c) $7.5 = (15 + 15e^{-0.2\times5})e^{-0.2T}$

$\Rightarrow \frac{7.5}{15(1+e^{-1})} = e^{-0.2T}$

$\Rightarrow -0.2T = \ln\frac{1}{2(1+e^{-1})}$

$\Rightarrow T = 5\ln\left(2 + \frac{2}{e}\right)$ hours