Nonlinear Ordinary Differential Equations, Jordan and Smith

Exercise 1.6

Find the equation of the limit cycle of

$$\ddot{x} + (4x^2 + \dot{x}^2 - 4)\dot{x} + 4x = 0.$$

What is its period?

$$h(x, \dot{x}) = (4x^2 + \dot{x}^2 - 4)\dot{x}$$

$$\frac{d\varepsilon}{dt} = -yh(x, \dot{x}) = -(4x^2 + \dot{x}^2 - 4)\dot{x}^2$$

$$\frac{d\varepsilon}{dt} = 0 \Rightarrow 4x^2 + \dot{x}^2 - 4 = 0$$

The equation of the limit cycle is $4x^2 + y^2 = 4$

and the period is $4 \int_0^1 \frac{dx}{\sqrt{4-4x^2}} = 2 \int_0^1 \frac{dx}{\sqrt{1-x^2}} = \pi.$