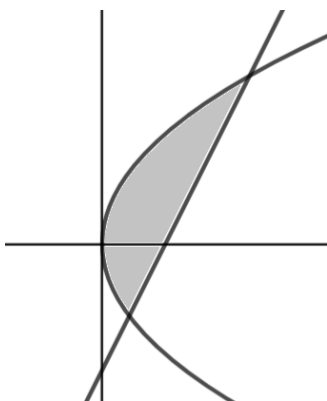The line with equation $8x - 4y - 21 = 0$ intersects the parabola C with equation $y^2 = 8x$ at the points P and Q. Find the area of the region enclosed by the line and the curve.

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At the intersections

$$y^2 = 4y + 21 \Rightarrow y^2 - 4y - 21 = 0 \Rightarrow (y - 7)(y + 3) = 0 \Rightarrow y = -3 \text{ or } y = 7.$$

The required area is

$$\int_{-3}^{7} \left(\frac{y}{2} + \frac{21}{8} - \frac{y^2}{8} \right) dy = \left[\frac{y^2}{4} + \frac{21y}{8} - \frac{y^3}{24} \right]_{-3}^{7}$$

$$= \left(\frac{49}{4} + \frac{147}{8} - \frac{343}{24} \right) - \left(\frac{9}{4} - \frac{63}{8} + \frac{27}{24} \right)$$

$$= 10 + \frac{210}{8} - \frac{370}{24}$$

$$= \frac{240}{24} + \frac{630}{24} - \frac{370}{24}$$

$$= \frac{500}{24}$$

$$= \frac{125}{6}$$

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