If $x = \sec\theta + \tan\theta$ show that $\dfrac{1}{x} = \sec\theta - \tan\theta$.

$$\frac{1}{x} = \frac{1}{\sec\theta + \tan\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta(1 + \sin\theta)}$$

$$= \frac{1 - \sin^2\theta}{\cos\theta(1 + \sin\theta)}$$

$$= \frac{1 - \sin\theta}{\cos\theta}$$

$$= \sec\theta - \tan\theta$$

Show also that $x^2 + \dfrac{1}{x^2} + 2 = 4\sec^2\theta$.

$$x^2 + \frac{1}{x^2} + 2 = (\sec\theta + \tan\theta)^2 + (\sec\theta - \tan\theta)^2 + 2$$

$$= \sec^2\theta + \tan^2\theta + 2\,\sec\theta\tan\theta + \sec^2\theta + \tan^2\theta - 2\,\sec\theta\tan\theta + 2$$

$$= 2\sec^2\theta + 2\tan^2\theta + 2$$

$$= 4\sec^2\theta$$