Find $I = \int x \sin nx \, dx$

$$I = -\frac{x \cos nx}{n} + \frac{1}{n}\int \cos nx \, dx$$

$$= -\frac{x \cos nx}{n} + \frac{\sin nx}{n^2} + C$$

Find $I = \int_{-\pi}^{\pi} x \sin nx \, dx$

$$I = \left(-\frac{x \cos nx}{n} + \frac{\sin nx}{n^2}\right) - \left(-\frac{x \cos nx}{n} + \frac{\sin nx}{n^2}\right)$$

$$= \left(-\frac{\pi \cos n\pi}{n}\right) - \left(-\frac{-\pi \cos n\pi}{n}\right)$$

$$= -\frac{2\pi \cos n\pi}{n}$$

Find the fourier series for $f(x) = x$ defined on the interval $[-\pi, \pi]$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

where

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \sin nx \, dx$$

$$f(x) = \sum_{n=1}^{\infty} \frac{-2 \cos n\pi \sin nx}{n}$$

$$= \sum_{n=1}^{\infty} \frac{-2(-1)^n \sin nx}{n}$$

https://www.desmos.com/calculator/gqllnxhlrp