STEP I 1991 question 16

The probability that the toad crosses the road safely is

$$\sum_{n=0}^{\infty} 0.1 \times 0.9^n = 1$$

The probability that the frog crosses safely is

$$0.1 +$$

$$0.9\left(0.1 + 0.9 \times \frac{1}{3} \times 0.2\right) +$$

$$0.9^2 \times \frac{2}{3}\left(0.1 + 0.9 \times \frac{2}{3} \times 0.2\right) +$$

$$0.9^3 \times \frac{2}{3} \times \frac{1}{3}(0.1 + 0.9 \times 0.2)$$

$$= 0.40816$$

The probability that the frog crosses safely, taking less time than the toad is

P(frog crosses at n=0 and toad crosses at n>0) + P(frog crosses at n=1 and toad crosses at n>1) + P(frog crosses at n=2 and toad crosses at n>2) + P(frog crosses at n=3 and toad crosses at n>3)

$$= 0.1 \times 0.9 +$$

$$0.9\left(0.1 + 0.9 \times \frac{1}{3} \times 0.2\right) \times 0.81 +$$

$$0.9^2 \times \frac{2}{3}\left(0.1 + 0.9 \times \frac{2}{3} \times 0.2\right) \times 0.729 +$$

$$0.9^3 \times \frac{2}{3} \times \frac{1}{3}(0.1 + 0.9 \times 0.2) \times 0.6561$$

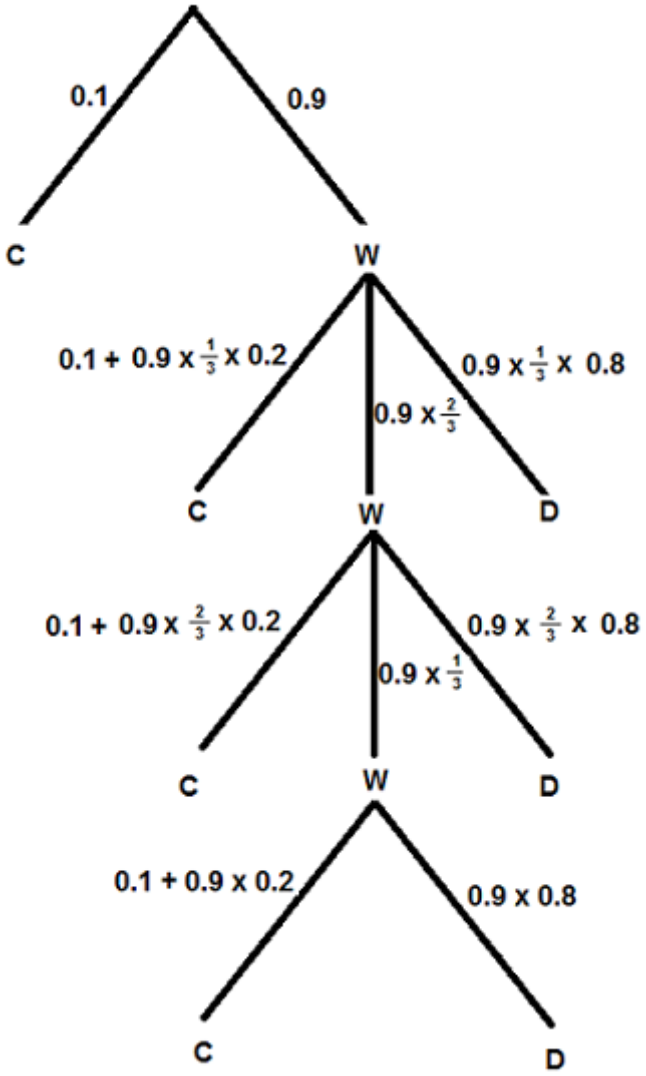
$$= 0.323005896$$

The probability that the frog is run over at some point if he has not arrived at the other side after two minutes is

$$\frac{P(frog\ is\ run\ over)}{1 - P(frog\ crosses\ safely\ in\ the\ first\ two\ minutes)}$$

$$= \frac{0.9^2 \times \frac{1}{3} \times 0.8 + 0.9^3 \times \left(\frac{2}{3}\right)^2 \times 0.8 + 0.9^4 \times \frac{2}{9} \times 0.8}{1 - \left\{0.1 + 0.9\left(0.1 + 0.9 \times \frac{1}{3} \times 0.2\right) + 0.9^2 \times \frac{2}{3}\left(0.1 + 0.9 \times \frac{2}{3} \times 0.2\right)\right\}}$$

$$= \frac{274}{295} = 0.9288\ correct\ to\ 4\ decimal\ places.$$


0.1
0.9
C
W
0.1 + 0.9 x 1/3 x 0.2
0.9 x 2/3
0.9 x 1/3 x 0.8
C
W
D
0.1 + 0.9 x 2/3 x 0.2
0.9 x 1/3
0.9 x 2/3 x 0.8
C
W
D
0.1 + 0.9 x 0.2
0.9 x 0.8
C
D